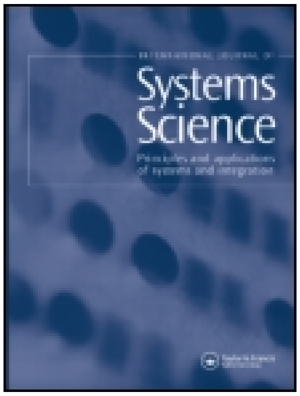

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Air-fuel ratio control in a gasoline engine

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The aim of this article is to design an air-fuel ratio control law for a gasoline IC engine. The air-fuel ratio is measured by a lambda sensor in the exhaust manifold. As a consequence, a variable transport delay arises in the model considered. A non-linear control approach based on a Takagi–Sugeno's model of the system is used. Then, two structures of control law are compared based on parallel distributed compensation control laws, which take into account the variable time delay. Finally, some simulations are given to show the efficiency of the developed control law.

Keywords: AFR control; Takagi–Sugeno's model; variable time delay; feed forward structure

AMS Subject Classifications: 37N35; 93C10; 93C42; 93C95

1. Introduction

The control of the air-fuel ratio (AFR) still remains a key point for engine control and especially for pollution emission purpose. The AFR is defined as the quantity of air over the quantity of fuel injected in each cylinder compared with stoichiometric condition. This variable characterises the quality of the combustion process and the performances of the engine according to fuel consumption and emissions. The catalytic technology for exhaust gas imposes an AFR of 1 ($\pm$5%). Generally, for a gasoline engine, the quantity of fuel injected in each cylinder controls the AFR.

The literature dealing with the AFR control is important. A first family of control laws is based on linear tools (Chang, Fekete, Amstutz and Powell 1995; Guzzella, Simons and Geering 1997; Kiencke and Nielsen 2000; Muske and Jones 2006). For example, in Guzzella et al. (1997) a feedback linearisation is developed and then a robust controller is designed. The main drawbacks of their approach are a lack of robustness according to noise measurements and the time delay introduced by the lambda sensor is not taken into account. In Kiencke and Nielsen (2000), the lambda sensor is taken into account in the computation of the control law. A PI controller is used to regulate the AFR in a neighbourhood of the desired value for fixed operating points of the engine. The efficiency is then limited for fast transient responses of the throttle valve. A second family deals with non-linear tools. For example, in Li and Yurkovich (1999) and Souder and Hedrick (2004) adaptive sliding mode control is used. In these approaches, the control law design does not take into account the time delay. In fact, the robustness of the approach is supposed to compensate this drawback. Their objective is to ensure a fast AFR regulation around 1 even during fast transient phases. Another proposed solution uses an accurate estimation of the AFR for each cylinder (Powell, Fekete and Chang 1998; Arsie, Pianese, Rizzo and Cioffi 2003; Benvenuti, Di Benedetto, Di Gennarob and Sangiovanni-Vincentellic 2003; Chauvin, Corde, Petit and Rouchon 2007), which allows an AFR regulation cylinder by cylinder. Of course, to derive such control laws accurate models are required. Moreover, for real-time control purpose, due to the sensors only a global AFR measurement is available. Therefore, the AFR estimation for each cylinder must use this single measurement and the performances can seriously decrease.

According to these preliminary remarks, the goal of this article is to propose a control law design that is able to cope with the time delay. This variable is defined as a function of the engine speed. Some performances are added, such as ensuring a good AFR regulation even during fast transient of the throttle valve. To achieve this goal, the first part of this article presents the proposed model: the well-known mean value engine model (Heywood 1988; Hendricks 2001). The global flow through the cylinders is considered

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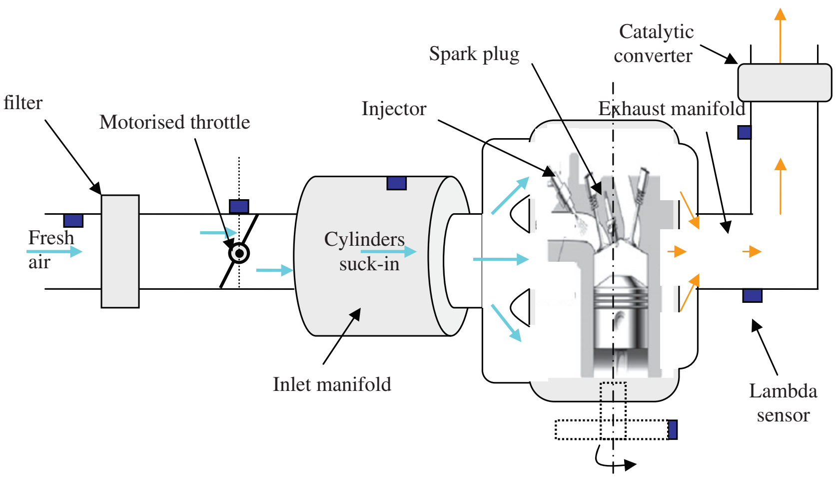

Figure 1. Air-fuel path in a classical gasoline engine.

and the same control is applied to each cylinder. The second part deals with the design of control laws. They are based on Takagi–Sugeno (TS) modelling (Takagi and Sugeno 1985). This modelling was described and applied to the engine in previous works (Lauber, Khiar and Guerra 2007, Khiar et al. 2007). Therein, the control laws are based on a state feedback and derived according to some recent theoretical results. The conditions obtained are sufficient, but use the Linear Matrix Inequality (LMI) constraints (Boyd et al. 1994). Lastly, some simulation results are given to compare the control laws and to show their efficiency.

2. Engine model for AFR control

The considered engine is a classical four-cylinder gasoline engine (Figure 1). The AFR control model is split into three parts: the cylinder mass airflow (quantity of air), the fuel mass flow dynamics (quantity of fuel) and the sensor model. For engine control purpose, the well-known mean value engine model (MVEM; Heywood 1988; Hendricks 2001) is used.

The MVEM is perfectly adapted to IC engine control design. It represents the global evolution of the main variables for an IC engine (such as manifold pressure, engine speed, AFR, etc.) using some mean value models. The main advantage of that kind of representation is the sample time of the simulation which is compatible with a real-time application. One limitation of the model is that some effects are neglected for the sake of simplicity, for example, the acoustic phenomena in the manifold and in the cylinders. Moreover, others are reduced to simpler models, such as, for example, the combustion part which is only considered as a time delay. Nevertheless, the MVEM allows coping with the dynamics of the variables involved in the control law design (manifold pressure, engine speed, etc.) with a good accuracy for a wide range of operating points (e.g. from 1000 to 5000 rpm for the engine speed). So, in the sequel, we focus on a range of operating points for which the MVEM is valid.

2.1. *Cylinder mass air flow*

The cylinder flow can be described by the speed density equation

$$\dot{m}_{\mathrm{cyl}}(N_{\mathrm{e}}, p_{\mathrm{im}}) = (\sigma_1 p_{\mathrm{im}} + \sigma_2)\frac{V_{\mathrm{d}}}{120 R T_{\mathrm{im}}} N_{\mathrm{e}}, \tag{1}$$

where p_{im} is the intake manifold pressure [Pa], T_{im} the intake manifold temperature [°K], V_{d} the volume displacement [m^3], R the constant for perfect gas [kJ/kgK] and N_{e} the engine speed [rpm]. This relation gives an estimation of the cylinder air mass flow through cylinders where σ_1 and σ_2 are two constant parameters.

2.2. *Fuel mass flow dynamic*

The fuel mass flow dynamic is described by a non-linear model. For indirect injection gasoline engine, the main phenomenon appearing in the fuel mass flow dynamic is a residual fuel film on the inlet pipe of each cylinder as shown in Figure 2.

A frequently used state space representation is given in Aquino (1989). The dynamic of the percentage of liquid fuel injected, χ, is described by a first-order model:

$$\begin{cases} \dfrac{\mathrm{d}\dot{m}_{\mathrm{ff}}(t)}{\mathrm{d}t} = \dfrac{1}{\tau_{\mathrm{f}}}(-\dot{m}_{\mathrm{ff}}(t) + \chi\,\dot{m}_{\mathrm{fi}}(t)) \\ \dot{m}_{\mathrm{fv}} = (1-\chi)\dot{m}_{\mathrm{fi}}(t) \\ \dot{m}_{\mathrm{f}}(t) = \dot{m}_{\mathrm{fv}}(t) + \dot{m}_{\mathrm{ff}}(t), \end{cases} \tag{2}$$

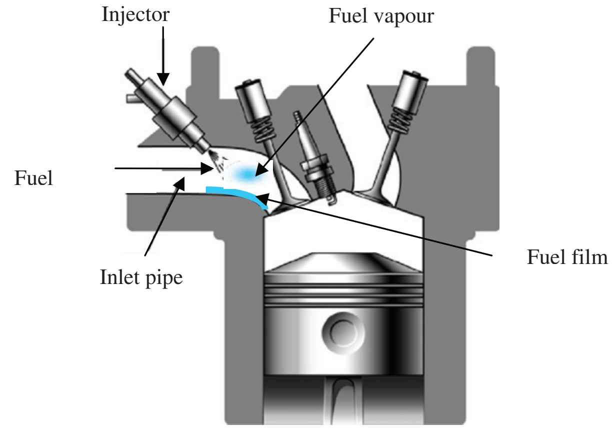

Figure 2. Fuel film phenomenon.

where τ_f is the constant time of the fuel vapour process [s], $\dot{m}_f(t)$ the mass fuel flow into the cylinders [kg/s], $\dot{m}_{ff}(t)$ the liquid mass fuel flow [kg/s], $\dot{m}_{fv}$ the vapour mass fuel flow [kg/s] and $\dot{m}_{fi}(t)$ the mass fuel flow from the injectors [kg/s].

In order to obtain a more accurate model, it is possible to consider χ as a variable depending on the throttle opening (Hendricks and Sorensen 1990) or on the engine speed N_e (Hou 2006). The second solution has been chosen in our case:

$$\tau_f = \sigma_3 N_e^{-\sigma_4} \tag{3}$$

$$\chi = \sigma_5 + \sigma_6 N_e, \tag{4}$$

where σ_3–σ_6 are constant parameters.

The model of the injector is given using a linear relationship between the mass fuel flow from the injectors and the injection time t_{inj} (the control variable) is given by the following (Powell et al. 1998; Kiencke and Nielsen 2000):

$$\dot{m}_{fi} = \frac{N_e}{30} k_{inj}\left(t_{inj} - t_0\right) \tag{5}$$

where t_0 is the injector dead time and k_{inj} a constant parameter characteristic of the injector.

2.3. *AFR model*

The AFR in the cylinders $\lambda_{cyl}(t)$ is defined by

$$\lambda_{cyl}(t) = \frac{m_{cyl}(t)}{\lambda_s m_f(t)}, \tag{6}$$

with $\lambda_s = 14.67$ the air-fuel stoichiometric ratio. So the time derivative of $\lambda_{cyl}(t)$ is given by:

$$\dot{\lambda}_{cyl}(t) = \left(\frac{1}{\lambda_s}\frac{\dot{m}_{cyl}(t)m_f(t)}{m_f(t)^2} - \frac{\dot{m}_f(t)}{m_f(t)}\lambda_{cyl}(t)\right). \tag{7}$$

Using the mean value model for a four-cylinder engine:

$$\dot{m}_f(t) = \frac{N_e}{30} m_f(t). \tag{8}$$

So, (7) becomes:

$$\dot{\lambda}_{cyl}(t) = \frac{N_e}{30}\left(\frac{1}{\lambda_s}\frac{\dot{m}_{cyl}(t)}{\dot{m}_f(t)} - \lambda_{cyl}(t)\right). \tag{9}$$

A variable time delay due to the transport of the gas flow from the cylinder to the lambda sensor is present in the model. This time delay depends on the engine speed, N_e. In this article, a classical UEGO sensor is considered to measure the AFR. A first order, as in Kim, Rizzoni and Utkin (2001), is chosen to represent the dynamic of the lambda sensor:

$$\dot{\lambda}_{mes}(t) = \frac{1}{\tau_\lambda}\left(-\lambda_{mes}(t) + \lambda_{cyl}(t - \tau(N_e))\right). \tag{10}$$

An approximation of the time delay $\tau(N_e)$ is given by Hendricks and Luther (2001):

$$\tau(N_e) = \frac{60}{N_e}\left(1 + \frac{1}{n_{cyl}}\right) \tag{11}$$

with n_{cyl} as the number of cylinders.

As the constant time τ_f varies from 6.6 to 18.7 ms for the engine speed varying from 1000 to 5000 rpm, and the lambda sensor has a time constant of about 0.1 s, we can neglect τ_f for the control design which leads us to simplify (2) into:

$$\dot{m}_f(t) = \dot{m}_{fi}(t). \tag{12}$$

To summarise, using the differential equations (9) and (10), and using the relations (1), (3), (4) and (5) a state-space representation can be written as:

$$\begin{cases} \dot{x}_1(t) = -f_1(t)x_1(t) + \dfrac{1}{\lambda_s} f_2(t)u(t) \\ \dot{x}_2(t) = \dfrac{1}{\tau_\lambda}(-x_2(t) + x_1(t - \tau(N_e))) \end{cases} \tag{13}$$

or equivalently

$$\dot{x}(t) = \begin{bmatrix} -f_1(N_e) & 0 \\ 0 & -\dfrac{1}{\tau_\lambda} \end{bmatrix} x(t) + \begin{bmatrix} 0 & 0 \\ \dfrac{1}{\tau_\lambda} & 0 \end{bmatrix} x(t - \tau(N_e)) + \begin{bmatrix} \dfrac{1\lambda_s}{f\ _2}(N_e) \\ 0 \end{bmatrix} u(t) \tag{14}$$

with $x_1(t) = \lambda_{cyl}(t)$ and where the bounded non-linear functions can be defined as:

$$f_1(t) = \frac{N_e}{30} \tag{15}$$

$$f_2(t) = \frac{N_e}{30}\dot{m}_{cyl}(t) \tag{16}$$

and the control variable as:

$$u(t) = \frac{1}{\dot{m}_f} \tag{17}$$

which allows coming back to the injection control time by neglecting the fast dynamic of (2):

$$t_{inj} = t_0 + \frac{30}{N_e k_{inj}} \frac{1}{u(t)}. \tag{18}$$

The relationship always exists because $u(t) \neq 0$ in a real context.

The output is then : $y(t) = x_2(t) = \lambda_{mes}(t)$. (19)

In the sequel, the design of control laws for the AFR using recent works on TS models with variable time delay is developed. The main advantage of the proposed approach is the systematic aspect of the design for non-linear system.

3. Control law design

We have defined few notations in the following. Let us consider a vector $z(t) \in \mathbb{R}^p$, scalar functions $h_i(z(t)) \geq 0$ satisfying the convex sum property: $\Sigma_{i=1}^r h_i(z(t)) = 1$. Considering Y_i, Y_{ij} $i, j \in \{1, \ldots, r\}$ matrices of appropriate dimension, we note: $Y_z = \Sigma_{i=1}^r h_i(z(t)) Y_i$ and $Y_{zz} = \Sigma_{i=1}^r \Sigma_{j=1}^r h_i(z(t)) h_j(z(t)) Y_{ij}$. For a square matrix X, the property of congruence with a row full rank matrix Π of appropriate dimension is defined using: $\Pi X \Pi^T$.

Lemma 1 (De Oliveira et al. 1999) (Extension of the Schur's complement): *With $P = P^T > 0$, $R > 0$ and X matrices of appropriate dimensions, the two following statements are equivalent*:

(i) $$X^T R X - P \leq 0 \tag{20}$$

(ii) there exists a matrix G such that

$$\begin{bmatrix} -P & X^T G^T \\ GX & R - G - G^T \end{bmatrix} \leq 0. \tag{21}$$

3.1. *TS modelling*

First of all, let us recall that TS fuzzy models allow representing exactly non-linear models in a compact set of the state variables (Tanaka and Wang 2001). A way to derive such TS models is to use the so-called sector non-linearity approach. It consists in representing a bounded non-linearity, i.e. $\underline{f_i} \leq f_i(\cdot) \leq \bar{f_i}$ using two functions verifying the convex sum property:

$$f_i(\cdot) = \underline{f_i} \cdot \frac{\bar{f_i} - f_i(\cdot)}{\bar{f_i} - \underline{f_i}} + \bar{f_i} \cdot \frac{f_i(\cdot) - \underline{f_i}}{\bar{f_i} - \underline{f_i}}. \tag{22}$$

Thus, considering the bounded function (15), a TS model composed of four linear models can be obtained. It corresponds to:

$$A_1 = A_2 = \begin{bmatrix} -\underline{f_1} & 0 \\ 0 & -\frac{1}{\tau_\lambda} \end{bmatrix}, \quad A_3 = A_4 = \begin{bmatrix} -\bar{f_1} & 0 \\ 0 & -\frac{1}{\tau_\lambda} \end{bmatrix},$$

$$D_1 = D_2 = D_3 = D_4 = \begin{bmatrix} 0 & 0 \\ \frac{1}{\tau_\lambda} & 0 \end{bmatrix},$$

$$B_1 = B_3 = \begin{bmatrix} \frac{1}{\lambda_s} \underline{f_2} \\ 0 \end{bmatrix}, \quad B_2 = B_4 = \begin{bmatrix} \frac{1}{\lambda_s} \bar{f_2} \\ 0 \end{bmatrix},$$

and a common matrix $C = [0 \quad 1]$.

The non-linear interpolating functions h_i, $i \in \{1, \ldots, 4\}$ are defined as:

$$h_1 = \frac{\bar{f_1} - f_1(\cdot)}{\bar{f_1} - \underline{f_1}} \cdot \frac{\bar{f_2} - f_2(\cdot)}{\bar{f_2} - \underline{f_2}}, \quad h_2 = \frac{\bar{f_1} - f_1(\cdot)}{\bar{f_1} - \underline{f_1}} \cdot \frac{f_2(\cdot) - \underline{f_2}}{\bar{f_2} - \underline{f_2}}$$

$$h_3 = \frac{f_1(\cdot) - \underline{f_1}}{\bar{f_1} - \underline{f_1}} \cdot \frac{\bar{f_2} - f_2(\cdot)}{\bar{f_2} - \underline{f_2}} \text{ and } h_4 = \frac{f_1(\cdot) - \underline{f_1}}{\bar{f_1} - \underline{f_1}} \cdot \frac{f_2(\cdot) - \underline{f_2}}{\bar{f_2} - \underline{f_2}}.$$

At last, we recall the classical control law used, the so-called Parallel Distributed Compensation (PDC; Tanaka and Wang 2001):

$$u(t) = -\sum_{i=1}^{r} h_i(z(t)) F_i x(t) = -F_z x(t). \tag{23}$$

It shares the same non-linear functions h_i, $i \in \{1, \ldots, r\}$ as the TS model. Thus, the goal is to obtain the gains F_i, $i \in \{1, \ldots, r\}$ according to prescribed specifications. The conditions are often derived as Linear Matrix Inequalities (LMI; Boyd et al. 1994) in order to be easily computed. The number of LMI conditions and/or variables can be quickly not tractable for the solvers: some solutions in reducing the size of the problems can be found in Delmotte, Guerra and Ksontini (2007).

3.2. *State feedback design*

To derive a control law, we consider the following family of TS model with time-varying state delay:

$$\begin{cases} \dot{x}(t) = A_z x(t) + D_z x(t - \tau(t)) + B_z u(t) \\ y(t) = C_z x(t) \end{cases} \tag{24}$$

with $\tau(t)$ as a bounded time-varying delay ($\tau(t) \leq \tau$).

Concerning such models, many works have been published recently. Pioneering works used constant time delay (Cao and Frank 2000), while recent works focus on TS model with variable delays. According to the hypothesis, two families of results can be distinguished. The former gives results that are delay independent (Wang 2004; Wang, Lin and Wang 2004). The latter is more interesting while dealing with delay-dependent conditions. It can be achieved considering the existence of a time delay derivative (Guan and Chen 2004) or using only the bounds of the delays (Lin, Wang and Lee 2006; Tian and Peng 2006).

Among these results, we decided to use the one of Lin et al. (2006). It represents a good compromise between complexity of the problem (in terms of additional variables and LMI constraints number) and pessimism of the obtained solutions. It is recalled in the following theorem, where (*) is used to specify a symmetric term.

Theorem 1 (Lin et al. 2006): *The TS model* (24) *can be stabilised via the PDC control law* (23) *if there exit matrices* $X>0$, $U_i>0$, $V_i>0$ *and* N_i $i\in\{1,\dots,r\}$, *such that the following LMI conditions hold for* $i,j\in\{1,\dots,r\}$, $i<j$:

$$\Upsilon_{ii}<0 \tag{25}$$

$$\Upsilon_{ij}+\Upsilon_{ji}<0 \tag{26}$$

$$\begin{bmatrix} -X & (*) \\ A_iX-B_iN_j & -U_i \end{bmatrix}\le 0, \quad \begin{bmatrix} -X & (*) \\ D_jX & -V_i \end{bmatrix}\le 0 \tag{27}$$

with $\Upsilon_{ij}=(A_i+D_i)X+X(A_i^T+D_i^T)-B_iN_j-N_j^TB_i^T+\tau D_i(U_i+V_i)D_i^T+2\tau X$.

Moreover, if the LMI problem has a solution, the control gains are given by $F_i=N_iX^{-1}$, $i\in\{1,\dots,r\}$.

The main idea of the present work is to use the combination of two control laws:

$$u(t)=u_1(t)+u_2(t). \tag{28}$$

The first one $u_1(t)$, a feedforward control, is derived according to the model without delays, i.e. considering the same matrices as (24) and defining a new state vector $\alpha(t)$ and a new output $y_1(t)$:

$$\begin{cases} \dot{\alpha}(t)=(A_z+D_z)\alpha(t)+B_zu_1(t) \\ y_1(t)=C_z\alpha(t). \end{cases} \tag{29}$$

Then, a classical design of a PDC law $u_1(t)=-L_z\alpha(t)$ can be done using the desired specifications and previous existing results, for example using pole placement for each linear model and verifying the stability *a posteriori*. The second control law $u_2(t)=-F_z(x(t)-\alpha(t))$ is supposed to cope with the time-varying delay. This gains F_i, and $i\in\{1,\dots,r\}$ are obtained after the design of $u_1(t)$ applying Theorem 1.

Lastly, if the state vector is not available, an observer is added to the structure to derive an output feedback law. Several designs are possible whether the premise variables are measured or not (Tanaka and Wang 2001; Sala, Guerra and Babuska 2005; Guerra, Kruszewski, Vermeiren and Tirmant 2006).

3.3. *Observer design*

We suppose that the variation of the time delay is known according to the relation (11). Then, we consider the following observer:

$$\begin{cases} \dot{\hat{x}}(t)=A_z\hat{x}(t)+D_z\hat{x}(t-\tau(t))+B_zu(t)+K_z(y(t)-\hat{y}(t)) \\ \hat{y}(t)=C_z\hat{x}(t). \end{cases} \tag{30}$$

Let us note the state estimation error $\tilde{x}(t)=x(t)-\hat{x}(t)$, then:

$$\dot{\tilde{x}}(t)=(A_z-K_zC_z)\tilde{x}(t)+D_z\tilde{x}(t-\tau(t)) \tag{31}$$

and define the following quantities:

$$\alpha_{ij}=\begin{bmatrix} P(A_i+D_i)+\left(A_i^T+D_i^T\right)P \\ -M_jC_i-C_i^TM_j^T+2\tau P & (*) & (*) \\ \tau D_i^TP & -\tau U_i & 0 \\ \tau D_i^TP & 0 & -\tau V_i \end{bmatrix} \tag{32}$$

$$\beta_{ij}=\begin{bmatrix} -P & (*) \\ PA_i-M_jC_i & U_i-2P \end{bmatrix}, \quad \gamma_{ij}=\begin{bmatrix} -P & (*) \\ G_jD_i & V_i-G_j-G_j^T \end{bmatrix}. \tag{33}$$

Theorem 2: *Consider the TS fuzzy model with variable time delay* (24), *and the associated fuzzy observer* (30) *and the* α_{ij}, β_{ij} *and* γ_{ij} *defined in* (32) *and* (33). *The equilibrium point of the state estimation error* (31) *is globally asymptotically stable if there exist matrices* $P>0$, $U_i>0$, $V_i>0$, G_i *and* M_i $i\in\{1,\dots,r\}$, *such that the following LMI conditions hold for* $i,j\in\{1,\dots,r\}$, $i\neq j$:

$$\alpha_{ii}<0, \quad \beta_{ii}<0, \quad \gamma_{ii}<0 \tag{34}$$

$$\frac{2}{r-1}\alpha_{ii}+\alpha_{ij}+\alpha_{ji}<0, \quad \frac{2}{r-1}\beta_{ii}+\beta_{ij}+\beta_{ji}<0, \quad \frac{2}{r-1}\gamma_{ii}+\gamma_{ij}+\gamma_{ji}<0. \tag{35}$$

Moreover, if the LMI problem has a solution, the observer gains are given by: $K_i=P^{-1}M_i$, $i\in\{1,\dots,r\}$.

Proof: Let us consider a quadratic Lyapunov function with $P>0$:

$$V(\tilde{x}(t))=\tilde{x}(t)^TP\tilde{x}(t). \tag{36}$$

Rewriting (31) using the classical transformation $\tilde{x}(t)-\tilde{x}(t-\tau(t))=\int_{t-\tau(t)}^{t}\dot{\tilde{x}}(s)\mathrm{d}s$ gives:

$$\dot{\tilde{x}}(t)=(A_z+D_z-K_zC_z)\tilde{x}(t)-D_z\int_{t-\tau(t)}^{t}\dot{\tilde{x}}(s)\mathrm{d}s. \quad (37)$$

Then, the derivative of $V(x(t))$ along the trajectories of (37) is:

$$\dot{V}(\tilde{x}(t))=2\tilde{x}(t)^TP\dot{\tilde{x}}(t)=2\tilde{x}(t)^TP(A_z+D_z-K_zC_z)\tilde{x}(t) - 2\tilde{x}(t)^TPD_z\int_{t-\tau(t)}^{t}\dot{\tilde{x}}(s)\mathrm{d}s. \quad (38)$$

Or equivalently:

$$\begin{aligned}\dot{V}(\tilde{x}(t))&=2\tilde{x}(t)^TP(A_z+D_z-K_zC_z)\tilde{x}(t)\\&\quad-\int_{t-\tau(t)}^{t}2\tilde{x}(t)^TPD_z\times[(A_z-K_zC_z)\tilde{x}(s)\\&\quad+D_z\tilde{x}(s-\tau(s))\mathrm{d}s.\end{aligned} \quad (39)$$

Then, the following very classical bounds are used introducing two matrices $U_z>0$ and $V_z>0$:

$$\begin{aligned}&-\int_{t-\tau(t)}^{t}2\tilde{x}(t)^TPD_z(A_z-K_zC_z)\tilde{x}(s)\,\mathrm{d}s\\&\leq\tau(t)\tilde{x}(t)^TPD_zU_z^{-1}D_z^TP\tilde{x}(t)\\&\quad+\int_{t-\tau(t)}^{t}\tilde{x}(s)^T\left(A_z^T-C_z^TK_z^T\right)U_z(A_z-K_zC_z)\tilde{x}(s)\,\mathrm{d}s\end{aligned} \quad (40)$$

$$\begin{aligned}&-\int_{t-\tau(t)}^{t}2\tilde{x}(t)^TPD_zD_z\tilde{x}(s-\tau(s))\mathrm{d}s\\&\leq\tau(t)\tilde{x}(t)^TPD_zV_z^{-1}D_z^TP\tilde{x}(t)\\&\quad+\int_{t-\tau(t)}^{t}\tilde{x}(s-\tau(s))^TD_z^TV_zD_z\tilde{x}(s-\tau(s))\,\mathrm{d}s.\end{aligned} \quad (41)$$

Applying these upper bounds to inequality (39) gives directly:

$$\begin{aligned}\dot{V}(\tilde{x}(t))&\leq2\tilde{x}(t)^TP(A_z+D_z-K_zC_z)\tilde{x}(t)\\&\quad+\tau(t)\tilde{x}(t)^TPD_z\left(U_z^{-1}+V_z^{-1}\right)D_z^TP\tilde{x}(t)\\&\quad+\int_{t-\tau(t)}^{t}\tilde{x}(s)^T\left(A_z^T-C_z^TK_z^T\right)U_z(A_z-K_zC_z)\tilde{x}(s)\mathrm{d}s\\&\quad+\int_{t-\tau(t)}^{t}\tilde{x}(s-\tau(s))^TD_z^TV_zD_z\tilde{x}(s-\tau(s))\,\mathrm{d}s.\end{aligned} \quad (42)$$

Let us use the Razumikhin's Theorem (Hale 1977). Suppose $V(\tilde{x}(t+\sigma))<(1+\delta)V(\tilde{x}(t))$ for all $\sigma\in[-\tau,0]$ with $\delta>0$. Assume for the moment that the following inequalities are satisfied:

$$\left(A_z^T-C_z^TK_z^T\right)U_z(A_z-K_zC_z)\leq P \quad (43)$$

$$D_z^TV_zD_z\leq P. \quad (44)$$

Then, from (42) we get

$$\begin{aligned}\dot{V}(\tilde{x}(t))&\leq2\tilde{x}(t)^TP(A_z+D_z-K_zC_z)\tilde{x}(t)\\&\quad+\tau(t)\tilde{x}(t)^TPD_z\left(U_z^{-1}+V_z^{-1}\right)D_z^TP\tilde{x}(t)\\&\quad+\tau(t)(1+\delta)\tilde{x}(t)^TP\tilde{x}(t)+\tau(t)\\&\quad\times(1+\delta)^2\tilde{x}(t)^TP\tilde{x}(t).\end{aligned} \quad (45)$$

So $\dot{V}(\tilde{x}(t))<0$ for sufficiently small $\delta>0$, if:

$$\begin{aligned}&P(A_z+D_z)+\left(A_z^T+D_z^T\right)P-PK_zC_z-C_z^TK_z^TP\\&\quad+2\tau P+\tau PD_z\left(U_z^{-1}+V_z^{-1}\right)D_z^TP<0.\end{aligned} \quad (46)$$

Then applying the Schur's complement, and using the bijective change of variable $M_z=PK_z$, (46) holds if:

$$\begin{bmatrix}\begin{matrix}P(A_z+D_z)+\left(A_z^T+D_z^T\right)P\\-M_zC_z-C_z^TM_z^T+2\tau P\end{matrix} & (*) & (*)\\ \tau D_z^TP & -\tau U_z & 0\\ \tau D_z^TP & 0 & -\tau V_z\end{bmatrix}<0. \quad (47)$$

Finally, the additional conditions (43) and (44) can be treated using Lemma 1. Introducing the additional variable G_z, (44) is equivalent to:

$$\begin{bmatrix}-P & (*)\\ G_zD_z & V_z-G_z-G_z^T\end{bmatrix}\leq0. \quad (48)$$

For (43), we fix $G=P$ in Lemma 1. Note that in that case the necessity is lost. Thus, a sufficient condition for (43) to hold is:

$$\begin{bmatrix}-P & (*)\\ PA_z-M_zC_z & U_z-2P\end{bmatrix}\leq0 \quad (49)$$

where (47), (48) and (49) give directly the expressions of α_{ij}, β_{ij}, γ_{ij} (32) and (33). Finally, conditions (34) and (35) of Theorem 2 are obtained using a relaxation due to Tuan, Apkarian, Narikiyo and Yamamoto (2001) and called parameterised linear matrix inequalities (PLMI). The proof is complete.

Remark 1: The main interest of the relaxation of Tuan et al. (2001) is that it does not introduce additional variables. Let us also point out that any kind of existing relaxation can be used in a similar way, for example, those of Kim and Lee (2000) or Liu and Zhang (2003). Nevertheless, the number of additional variables introduced can be very quickly incompatible with the actual LMI solvers. To conclude the global scheme control a schematic representation is given in Figure 3.

4. Application to SI engine

For simulation purposes, the constant parameters are chosen in Benvenuti et al. (2003) in order to be as close as possible from a 'real' engine. Indeed, in these references the parameters were identified on a test bench. The values used are given in Table 1.

To test the efficiency of the proposed method two structures of control law with and without the feedforward 'Fuzzy controller 1' (Figure 3) are compared together.

The gains obtained for the different steps of the proposed method are summed up here. The upper bound for the delay is: $\tau = 0.1$ s. The gains obtained for the feedforward control are:

$$L_1 = [33.5682 \quad 14.6876], \quad L_2 = [28.5779 \quad 8.4400],$$
$$L_3 = [31.8239 \quad 14.6503] \text{ and } L_4 = [31.8239 \quad 14.6503].$$

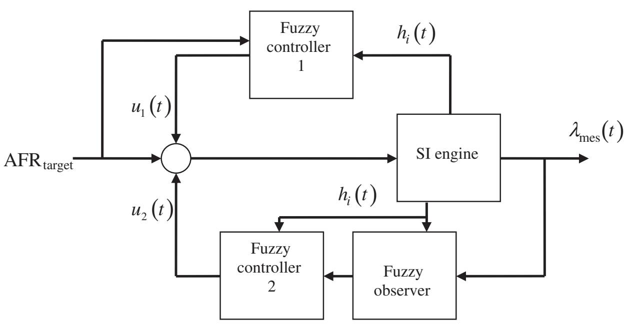

Figure 3. Scheme of the control law.

The gains for the PDC control law taking into account the time varying delay are:

$$F_1 = [-2.5294 \quad 0.0362], \quad F_2 = [-0.8878 \quad 0.0183],$$
$$F_3 = [-4.7840 \quad 0.0410] \text{ and } F_4 = [-2.8646 \quad 0.0193].$$

And finally, the gains of the observer are:

$$K_1 = [7.9619 \quad 15.2284], \quad K_2 = [7.9619 \quad 15.2284],$$
$$K_3 = [4.1968 \quad 13.1834] \text{ and } K_4 = [4.1968 \quad 13.1834].$$

The goal of the simulation is to track a reference AFR through the conditions below for the manifold pressure and the engine speed (Figure 4). For the chosen value the time delay varies around 0.05 s.

The Figures 5 and 6 show the results obtained with the control law without the feedforward structure. The control law allows following the desired AFR target. Nevertheless, the response time is slow, about 1 s, which is mainly due to the conservatism of the LMI conditions when the variable delay is taking into account. Better results are obtained adding the feed-forward structure as shown in Figures 7 and 8. Indeed, the response time is decreasing from 0.5 to 0.3 s.

5. Conclusion

The purpose of this work was to develop a control law for the regulation of the AFR of a spark ignition engine. First, the model equations were given and

Table 1. Parameters of the model.

Parameter	σ_1	σ_2	σ_3	σ_4	σ_5	σ_6	k_{inj}	t_0	τ_λ
Value	0.0281	−1.6208	1.67	0.65	9.6×10^{-5}	0.7236	1.93×10^{-3}	0.75×10^{-3}	0.1

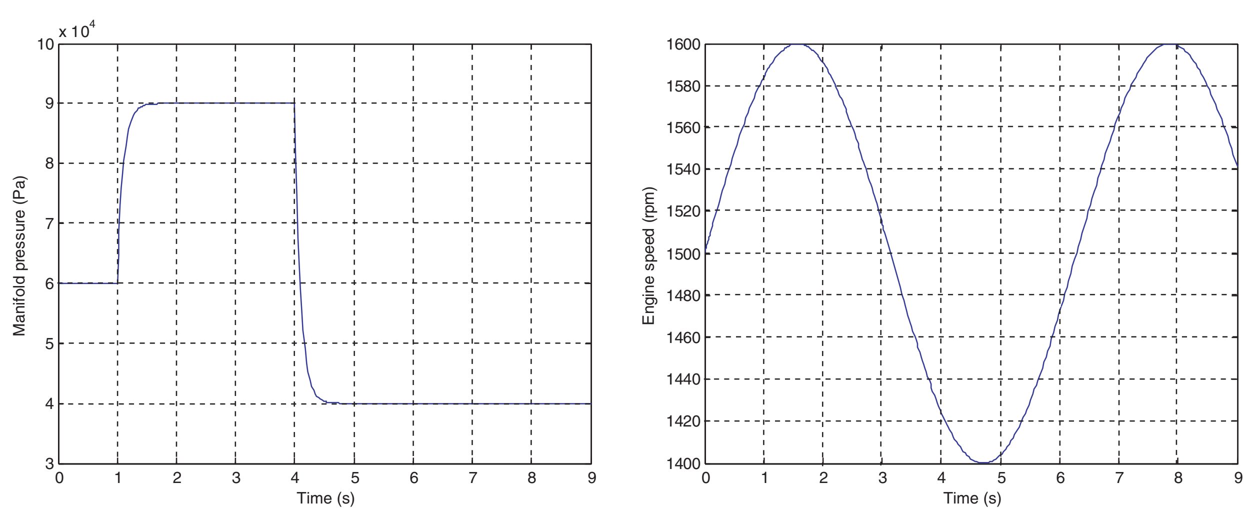

Figure 4. Manifold pressure and engine speed.

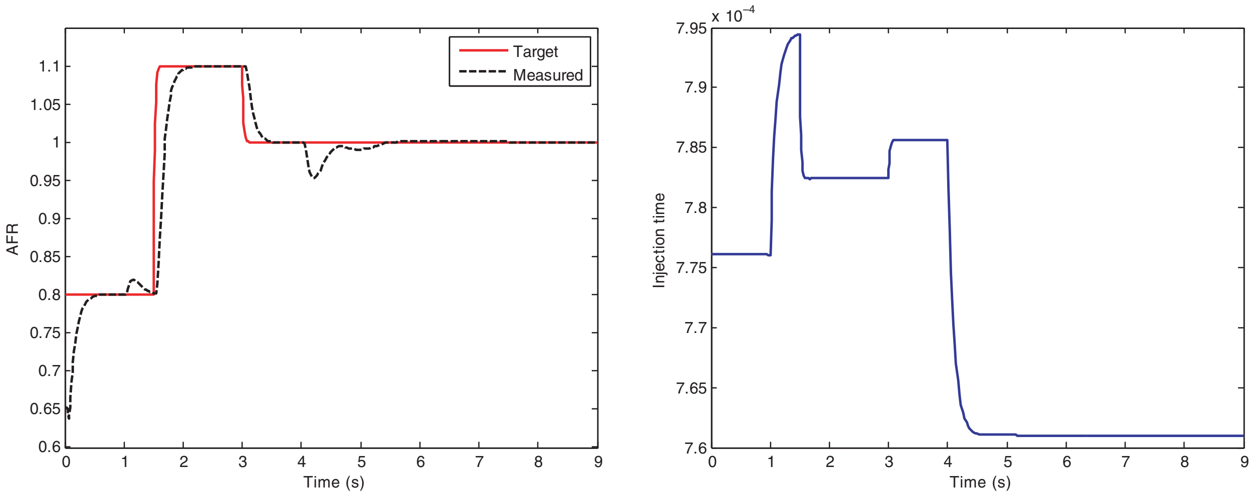

Figure 5. Tracking control of the AFR without feedforward structure and associated control.

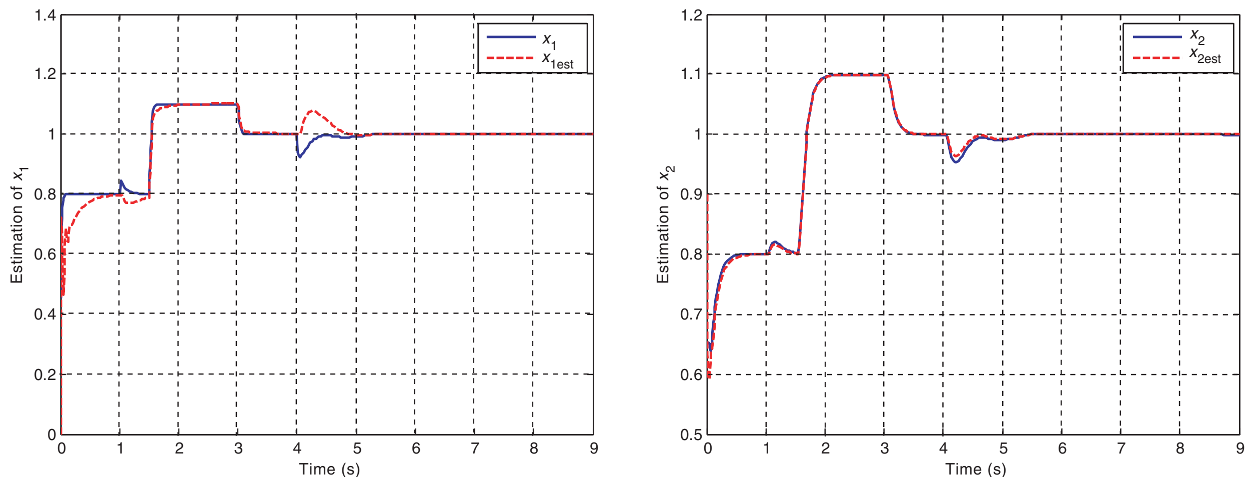

Figure 6. Real and estimated states.

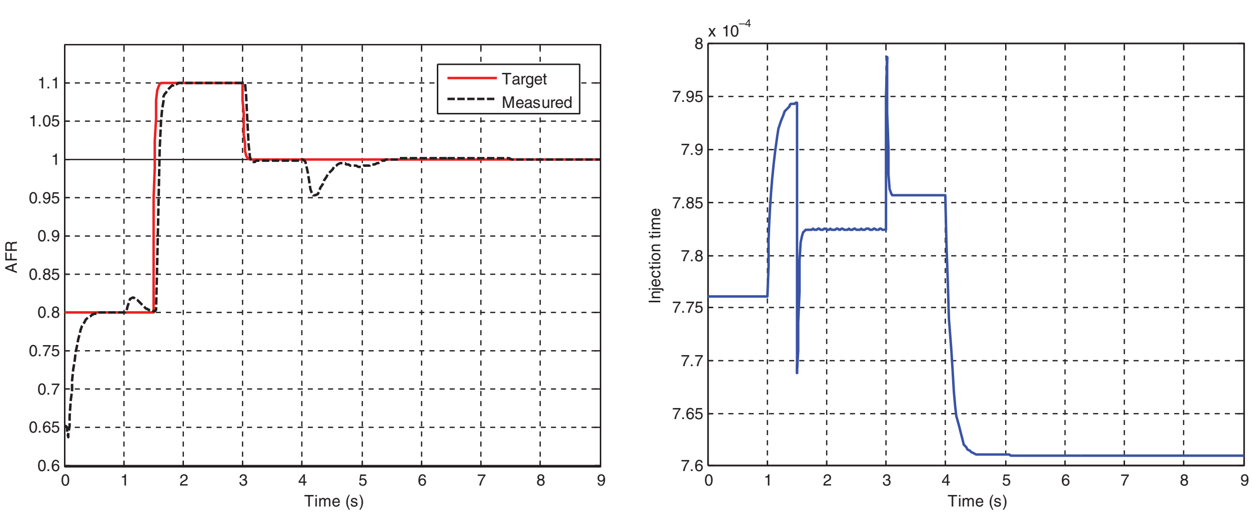

Figure 7. Tracking control of the AFR with feedforward structure and associated control.

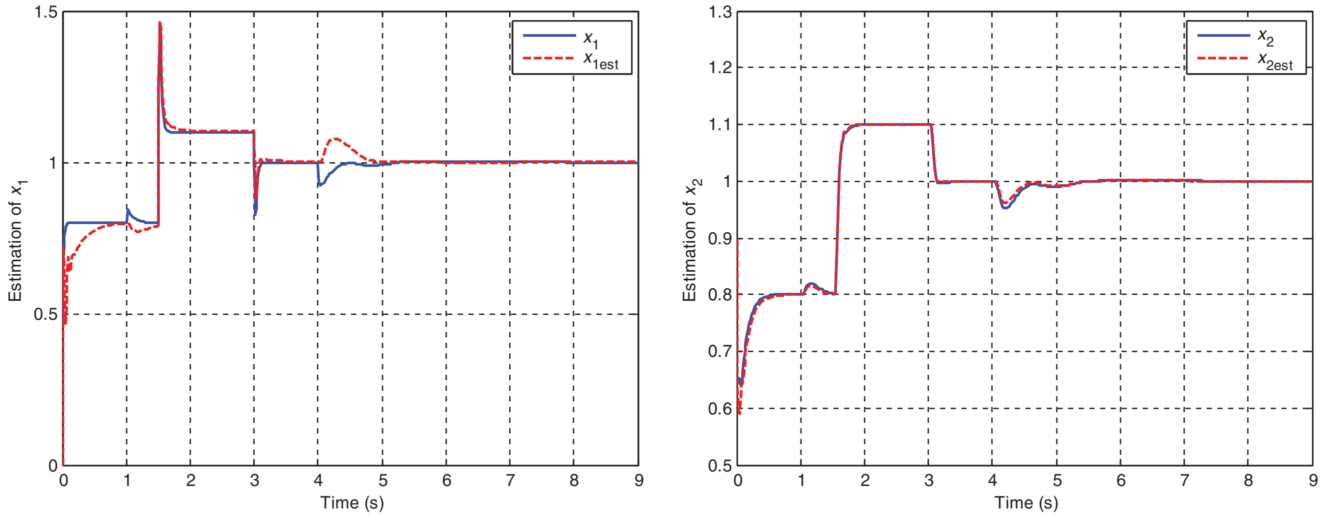

Figure 8. Real and estimated states.

formulated as a non-linear state space model. Then, some fuzzy TS control tools were developed. They are concerned with the conditions for stabilisation of a class of non-linear TS models with variable time delay. The controller was also designed with a feedforward structure to achieve performances in terms of response time. The results show the efficiency of the proposed method. The main advantage of this control design is its systematic aspect to deal with a large class of non-linear systems. Future works are concerning the extension of the proposed control approach to the case where each injector is considered independently, and so an estimation of the contribution of each cylinder to the AFR is needed.

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